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Magnification of Starting Torques of dc Motors by Maximum Power Point Trackers in Photovoltaic Systems

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MAGNIFICATION OF STARTING TORQUES OF DC MOTORS BY MAXIMUM POWER POINT TRACKERS IN PHOTOVOLTAIC SYSTEMS

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ABSTRACT

The paper deals with the calculation of the starting torque ratio of the permanent magnet, series, and shunt excited dc motors when powered by solar cell arrays for two cases: with and without a maximum power point tracker (MPPT). Defining a motor torque magnification by the ratio of the motor torque with an MPPT to the motor torque without an MPPT, one may get a magnification of 3 for the permanent magnet motor and a magnification of 7 for both the series and shunt motors. The study also shows that all motor types are less sensitive to solar insolation variation in systems including MPPT's as compared to systems without MPPT's.

INTRODUCTION

Direct current (dc) motors are used in photovoltaic (PV) drive systems [1-6], for example, in cooling application where the motors drive reciprocating vapor compressors, and in water-pumping systems for irrigation or water supply where motors drive positive displacement or centrifugal pumps. In a direct coupled (with no battery storage) PV system, the solar cell array is directly connected to the motor-load couple. These systems are relatively simple and inexpensive to operate. A direct coupled system may include a maximum power point tracker (MPPT) to improve its performance whenever it is needed [7].

The starting to rated torque ratio and the variation of the starting torque with insolation are important characteristics of a dc motor. For various dc motor types these values are different, and since dc motors may be used in different applications in PV systems, these characteristics are to be calculated by the PV system designer. The paper deals with the calculation of the starting torque for the permanent magnet, series and shunt excited dc motor for two cases: (1) when an MPPT is not included in the system and (2) when an MPPT is included in the system.

The calculation of the motor starting torque for the various motor types was made with some assumptions and approximations. Nevertheless, they are not too far off from real values and, therefore, can be used for comparison between the different motors in systems both with and without MPPT's. A main assumption is the linear dependence of the magnetic flux on the field current. Another assumption reflects the field and armature reactances. The torques would result in somewhat lower values without these assumptions.

MOTOR EQUATIONS

The circuit diagram of permanent magnet, series, and shunt excited motors are shown in Fig. 1(a) to (c), respectively. The motor voltage and torque equations are:

$$V_m = E + I_a R \tag{1}$$

$$E = k_e \phi n \tag{2}$$

$$T = k_T \phi I_a \tag{3}$$

where

V_m the motor applied voltage, V

E the motor-electro-motive force, e.m.f., V

I_a the motor armature current, A

R the motor armature circuit resistance, Ω

ϕ the motor flux, Wb

n the motor shaft speed, rpm

T the motor electromagnetic torque, N·m

k_e, k_T the motor voltage and torque constants, respectively

Assuming a linear dependence of the magnetic flux on the field current (a linear motor model) one can write the following relations:

(a) for the permanent magnetic motor;

$$\phi = C_1 \quad \text{and} \quad T = C_2 I_a \tag{4}$$

(b) for the series motor;

$$\phi = C_3 I_a, \quad T = C_4 I_a^2, \quad \text{and} \quad R = R_a + R_s \tag{5}$$

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(c) for the shunt motor;

$$\phi = C_5 I_f \quad \text{and} \quad T = C_6 I_a I_f \tag{6}$$

$$I_m = I_a + I_f \quad \text{and} \quad I_f = \frac{V_m}{R_{sh}} \tag{7}$$

$$T = C_7 I_a I_f \tag{8}$$

where

I_m the motor terminal current

I_f the shunt excited motor field current

R_s the series motor field resistance

R_{sh} the shunt excited motor field resistance

C_1-C_7 constants

For the permanent magnet and series excited motors we write:

$$I_m = I_a \tag{9}$$

By direct coupling the motor to the solar cells we have:

$$V_m = V \quad \text{and} \quad I_m = I \tag{10}$$

where V and I are the array voltage and current, respectively. For the purpose of comparing the different motor types, it is assumed that the rated armature voltage drop (including the voltage drop on the brushes) for all motor types is 10 percent of the rated motor terminal voltage. A good system design corresponds to rated motor operation V_n, I_n close to the maximum power point V_M, I_M of the solar cell array. The insolation level for rated motor operation is taken to be about 0.8 Sun for the appropriate array. The ratio of the short circuit current I_{sc} to the maximum power-current I_M of a typical solar cell array is about 1.2. This ratio; the armature voltage drop percentage; and the motor rated operation is used in the analysis for all motor types, i.e.,

$$I_M = I_n, \quad V_M = V_n, \quad \frac{I_n R}{V_n} = 0.1$$

and (11)

$$\frac{I_{sc}}{I_M} = 1.2$$

The system's operating point I_M, V_M is determined by the intersection of the I-V characteristics of the solar cell array with the I-V characteristic of the motor (Eq. (1)) as shown in Fig. 2. The slope of the motor characteristic is $\theta = \tan^{-1} 1/R$, and since the resistance of the armature circuit is low, the slope θ is large. At the instant of motor starting $n = 0$, therefore $E = 0$ and the motor characteristic is thus represented by a straight line (see Eq. (1)) with a slope of $\tan^{-1} 1/R$ passing through the origin as shown in Fig. 2. The starting current is approximately the short circuit current of the array, i.e.,

$$I_{st} \equiv I_{sc} \tag{12}$$

and the motor terminal voltage at starting is:

$$V_{st} \equiv I_{sc} R \tag{13}$$

MOTOR STARTING TORQUE RATIO

Permanent Magnet Motor

The motor starting current is:

$$I_{st} = I_a \equiv I_{sc}^{+} \tag{14}$$

and the starting current ratio becomes:

$$\frac{I_{st}}{I_n} = \frac{I_{sc}}{I_M} \tag{15}$$

where T_{st} and T_n are the starting and rated torques, respectively. This ratio is 1.2 (Eq. (11)) for the permanent magnet motor.

The motor starting torque ratio is given by Eqs. (4) and (15), i.e.,

$$\frac{T_{st}}{T_n} = \frac{I_{sc}}{I_M} \tag{16}$$

Series Excited Motor

The motor starting current ratio is:

$$\frac{I_{st}}{I_n} = \frac{I_{sc}}{I_M} \tag{17}$$

and the motor starting torque ratio is given by Eqs. (5) and (17):

$$\frac{T_{st}}{T_n} = \left(\frac{I_{sc}}{I_M}\right)^2 \tag{18}$$

This ratio (Eq. (11)) is 1.44 for the series excited motor.

Shunt Excited Motor

We shall first calculate the rated armature current and torque. At the maximum power point, the rated armature current according to Eq. (7) is:

$$I_a = I_M - I_f = I_M - \frac{V_M}{R_{sh}} \tag{19}$$

and the rated motor torque according to Eq. (8) is:

$$T_n = C_7 \left(I_M - \frac{V_M}{R_{sh}}\right) \frac{V_M}{R_{sh}} \tag{20}$$

+In the following equations an equal rather than an approximate sign is used.

At starting (E = 0), the motor is represented by two resistors connected in parallel: the armature R_a and field R_{sh} resistors, i.e., $R_a \| R_{sh}$. The motor terminal current is $I_m = I_{sc}$, therefore, the armature current at starting (according to the current dividing rule) is:

$$I_{a,st} = I_{sc} \frac{R_a \| R_{sh}}{R_a} \tag{21}$$

and the field current at starting is:

$$I_{f,st} = I_{sc} \frac{R_a \| R_{sh}}{R_{sh}} \tag{22}$$

The shunt motor starting torque according to Eq. (8) becomes:

$$T_{st} = C_7 I_{sc}^2 \frac{(R_a \| R_{sh})^2}{R_a R_{sh}} \tag{23}$$

and the motor starting torque ratio (Eqs. (20) and (23)) is:

$$\frac{T_{st}}{T_n} = \frac{I_{sc}^2 \frac{(R_a \| R_{sh})^2}{R_a R_{sh}}}{\left(I_M - \frac{V_M}{R_{sh}}\right)\frac{V_M}{R_{sh}}} \tag{24}$$

Equation (24) can be approximated by $I_M \gg I_f$ and $R_{sh} \gg R_a$ resulting in:

$$\frac{T_{st}}{T_n} = \left(\frac{I_{sc}}{I_M}\right)^2 \frac{I_M R_a}{V_M} \tag{25}$$

This ratio, according to Eq. (11) is 0.14, i.e., the starting torque ratio of the shunt motor is very low and is usually not sufficient to overcome the starting torque of the mechanism. This low value is attributed by the low field current at starting caused by the low voltage at the motor terminals (Eq. (13)).

THE MAGNIFICATION OF THE MOTOR STARTING TORQUE BY AN MPPT

By matching the solar cell array to the motor by means of a maximum power point tracker (MPPT), the motor operation can be improved. The MPPT consists of a power processing circuit, as Buck, Buck/Boost, or Boost [8] circuits, controlled by a signal circuit unit which drives the power processing circuit such that the solar cell array operates at its maximum power point, V_M and I_M. The power processing circuit of the MPPT can be modeled by a controlled time-variable-transformer (TVT) [9] in which the transformation ratio k is changed continuously, corresponding to variation in the load operating point. A system consisting of a solar cell array, an MPPT and a dc motor is shown in Fig. 3. The motor is represented by the e.m.f. E and the armature circuit resistance R; the TVT is assumed to be loss free, therefore all of the array power is delivered to the motor load.

The input/output equations of the TVT are:

$$V_M = kV_m \quad \text{and} \quad I_M = I_m/k \tag{26}$$

The motor voltage equation is:

$$V_m = E + I_m R \tag{27}$$

Using Eqs. (26) and (27) and solving for k we get:

$$k = \left[\frac{E}{2V_M} + \sqrt{\left(\frac{E}{2V_M}\right)^2 + \frac{I_M R}{V_M}}\right]^{-1} \tag{28}$$

At motor starting E = 0 and Eq. (28) reduces to:

$$k_{st} = \left(\frac{V_M}{I_M R}\right)^{1/2} = \left(\frac{P_M}{I_M^2 R}\right)^{1/2} \tag{29}$$

Using Eqs. (26) and (29), the motor starting current is:

$$I_{m,st} = \left(\frac{P_M}{R}\right)^{1/2} = \left(\frac{V_M I_M}{R}\right)^{1/2} \tag{30}$$

In the previous section we have calculated the starting torque ratios of the different dc motors when an MPPT was not included in the system. The starting torque is increased when an MPTT is included in the system, the amount of which depends on the motor type. The magnification of the starting torque will now be calculated. We define a torque magnification factor m_T by the ratio of the starting torque with an MPPT to the starting torque without an MPPT:

$$m_T = \frac{T_{st} \text{ with MPPT}}{T_{st} \text{ without MPPT}} \tag{31}$$

Permanent Magnet Motor

The motor starting torque is proportional to the motor starting current, therefore, from Eqs. (4), (11), (12), and (29), the torque magnification is:

$$m_T = \frac{\left(\frac{V_M I_M}{R}\right)^{1/2}}{I_{sc}} = \left(\frac{I_M}{I_{sc}}\right)\left(\frac{V_n}{I_n R}\right)^{1/2} = 2.64 \tag{32}$$

Series Excited Motor

The motor starting torque is proportional to the square of the armature starting current, therefore, from Eqs. (5), (11), (12), and (30), the torque magnification is:

$$m_T = \frac{\left(\frac{V_M I_M}{R}\right)}{I_{sc}^2} = \left(\frac{I_M}{I_{sc}}\right)^2 \left(\frac{V_n}{I_n R}\right) = 6.94 \qquad (33)$$

Shunt Excited Motor

At starting, the equivalent motor resistance is $R_a \| R_{sh}$. The motor starting current is given by Eq. (30), i.e.,

$$I_{m,st} = \left(\frac{P_M}{R_a \| R_{sh}}\right)^{1/2} \qquad (34)$$

According to the current dividing rule, the armature current at starting is:

$$I_{a,st} = I_{m,st} \frac{R_a \| R_{sh}}{R_a} = \left(\frac{P_M}{R_a \| R_{sh}}\right)^{1/2} \frac{R_a \| R_{sh}}{R_a} \qquad (35)$$

and the field current at starting is:

$$I_{f,st} = \left(\frac{P_M}{R_a \| R_{sh}}\right)^{1/2} \frac{R_a \| R_{sh}}{R_{sh}} \qquad (36)$$

The motor starting torque is proportional to the armature and the field currents, using Eqs. (11), (21), (22), (35), and (36), the torque magnification factor is:

$$m_T = \frac{P_M}{I_{sc}^2 (R_a \| R_{sh})} = \left(\frac{I_M}{I_{sc}}\right)^2 \frac{V_n}{I_n R_a} = 6.94 \qquad (37)$$

i.e., the same value as for the series motor.

VARIATION OF MOTOR STARTING TORQUE WITH INSOLATION

As the solar insolation varies during the day, the motor starting torque will vary accordingly. We will again distinguish between systems with and without MPPT's.

Motor Starting Torque Without An MPPT

At starting, the motor current is approximately the solar cell array short circuit current, Eq. (12), and since the array short circuit current is linearly proportional to the solar insolation, the motor starting current is thus also linearly proportional to the solar insolation:

$$\frac{I_{st}}{I_{st,r}} = \frac{I_{sc}}{I_{sc,r}} = \frac{S}{S_r} \qquad (38)$$

where S and S_r are an arbitrary and reference insolation, respectively, I_{sc} and $I_{sc,r}$ are the array short circuit currents corresponding to S and S_r, respectively; and r denotes reference. We define an insolation-starting-torque-factor $t(S)$ by the ratio of the motor starting torque at an arbitrary insolation S to the starting torque at a reference insolation S_r, i.e.,

$$t(S) = \frac{T_{st}(S)}{T_{st}(S_r)} \qquad (39)$$

The torque factor for the various motor types as function of the insolation are:

(a) Permanent magnet motor (Eqs. (4), (38), and (39))

$$t(S) = \left(\frac{S}{S_r}\right) \qquad (40)$$

(b) Series excited motor (Eqs. (5), (38), and (39))

$$t(S) = \left(\frac{S}{S_r}\right)^2 \qquad (41)$$

(c) Shunt excited motor (Eqs. (6), (38), and (39))

$$t(S) = \left(\frac{S}{S_r}\right)^2 \qquad (42)$$

The results show that the starting torque ratio of the permanent magnet motor is less sensitive to insolation variation than the series and shunt excited motors.

Motor Starting Torque with an MPPT

The operating points of a system including an MPPT with varying insolation are along the maximum power line of the solar cell array. The variation of the array voltage is approximately logarithmic dependent with insolation; and if we assume a constant motor voltage V_m, to some degree of accuracy, the motor starting current according to Eqs. (11) and (30) may be written as:

$$I_{m,st} = \left(\frac{V_M I_M}{R}\right)^{1/2} = C_8 I_{sc}^{1/2} = C_9 S^{1/2} \qquad (43)$$

and according to Eqs. (11), (35), and (36):

$$I_{a,st} = C_{10} S^{1/2} \quad \text{and} \quad I_{f,st} = C_{11} S^{1/2} \qquad (44)$$

where $C_8 - C_{11}$ are constants.

The torque factors for the various motor types in systems including MPPT's become:

(a) Permanent magnet motor (Eqs. (4), (39), and (43))

$$t(S) = \left(\frac{S}{S_r}\right)^{1/2} \qquad (45)$$

(b) Series excited motor (Eqs. (5), (39), and (43))

$$t(S) = \left(\frac{S}{S_r}\right) \tag{46}$$

(c) Shunt excited motor (Eqs. (6), (39), and (44))

$$t(S) = \left(\frac{S}{S_r}\right) \tag{47}$$

The results show that the starting torque of the permanent magnet motor is less sensitive to insolation variation than the series and shunt excited motors. Another important result for all motor types is that systems including MPPT's are less sensitive to insolation variation than systems without MPPT's. These can be seen by comparing Eq. (40) with (45), Eq. (41) with (46), and Eq. (42) with (47), for the permanent magnet, series and shunt excited motors, respectively.

CONCLUSIONS

The starting torque ratio and the variation of the starting torque with insolation of the permanent magnet, series and shunt excited dc motors powered by solar cell arrays were calculated for systems with and without a maximum-power-point tracker (MPPT). The starting torque magnification factor m_T was defined by the ratio of the starting torque of the motor with an MPPT to the starting torque without an MPPT. The results are summarized in Table I and shows that high magnification of the starting torque of dc motors is obtained in systems including MPPT's. Although the torque magnification of the shunt excited motor is high, the starting torque remains low, i.e., less than the rated torque. Another important result is the effect of insolation on the motor starting torque in systems including MPPT's. All motor types are less sensitive to solar insolation variation in systems including MPPT's as compared to systems without MPPT's. This result is summarized in Table II. The results of the present study were obtained for a linear motor model. The analysis presented may assist the PV system designer to determine the starting torques and their variation with insolation for the different dc motor types when MPPT's are included in the systems.

TABLE I. - STARTING TORQUE RATIO OF DC MOTORS

	Permanent magnet	Series	Shunt
Without MPPT	1.2	1.44	0.14
With MPPT	3.17	9.99	.97
Magnification	2.64	6.94	6.94

TABLE II. - INSOLATION STARTING TORQUE FACTOR OF DC MOTORS

	Permanent magnet	Series	Shunt
Without MPPT	(S/S_r)	$(S/S_r)^2$	$(S/S_r)^2$
With MPPT	$(S/S_r)^{1/2}$	(S/S_r)	(S/S_r)

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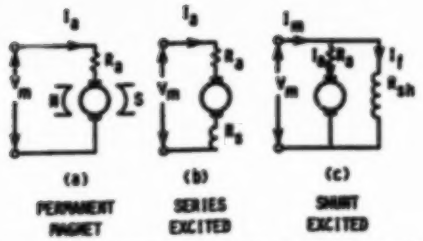

FIGURE 1. - CIRCUIT DIAGRAMS OF DIFFERENT TYPES OF dc MOTORS.

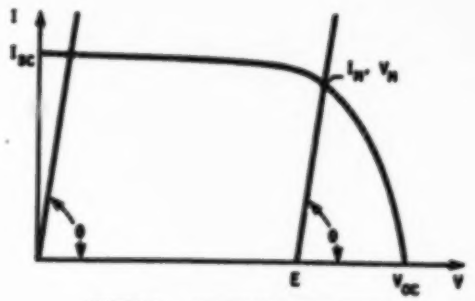

FIGURE 2. - SYSTEM OPERATING POINT.

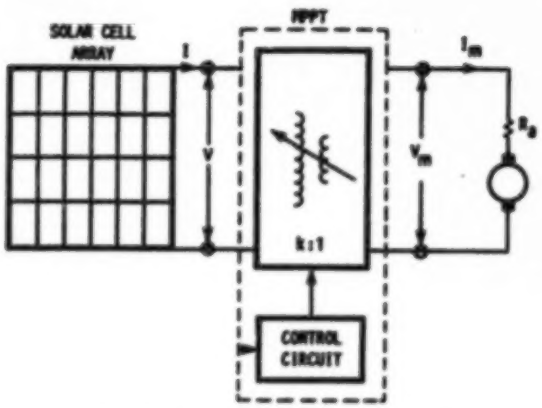

FIGURE 3. - SOLAR CELL SYSTEM WITH AN MPPT.

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16. Abstract

Direct current (dc) motors are used in terrestrial photovoltaic (PV) systems such as in water-pumping systems for irrigation and water supply. Direct current motors may also be used for space applications. Simple and low weight systems including dc motors may be of special interest in space where the motors are directly coupled to the solar cell array (with no storage). The system will operate only during times when sufficient insolation is available. An important performance characteristic of electric motors is the starting to rated torque ratio. Different types of dc motors have different starting torque ratios. These ratios are dictated by the size of solar cell array, and the developed motor torque may not be sufficient to overcome the load starting torque. By including a maximum power point tracker (MPPT) in the PV system, the starting to rated torque ratio will increase, the amount of which depends on the motor type. The paper deals with the calculation of the starting torque ratio of the permanent magnet, series and shunt excited dc motors when powered by solar cell arrays for two cases: with and without MPPT's. Defining a motor torque magnification by the ratio of the motor torque with an MPPT to the motor torque without an MPPT, one may get a magnification of 3 for the permanent magnet motor and a magnification of 7 for both the series and shunt motors. The study deals also with the effect of the variation of solar insolation on the motor starting torque. All motor types are less sensitive to insolation variation in systems including MPPT's as compared to systems without MPPT's. The analysis of this paper will assist the PV system designer to determine whether or not to include an MPPT in the system for a specific motor type.

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